Chapter 5

ENHANCING MATHEMATICAL PROBLEM-SOLVING EXPERIENCES THROUGH LEARNING ANALYTICS

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Abstract: Even with all the technological advancements, a majority of students in today's classes still do their mathematics exercises on paper. This approach does not provide teachers with much information about the way students worked on their exercises. Given this, how could teachers understand the problem-solving process undertaken by students? How could they know where in the process students struggled? How could they understand students' competence, confidence and metacognitive abilities? How could the teachers be given real-time information about the challenges faced by students? How could teachers consume this information and in a timely manner translate it into appropriate support and intervention to the students in need? MATHeX is a learning analytics tool that aims to capture as much real-time data about students' work as possible to better understand their study processes and the relation between study processes and learning outcomes. Tracking such data allows for an in-depth analysis of competence, confidence, and targeted metacognitive abilities of students in underlying mathematical concepts as well as a wider view of their study behavior. These data can then be analyzed, transformed and displayed in an interactive dashboard for the consumption of students, teachers, parents, and school administrators. Would learning analytics be able to enhance the role and effectiveness of a human tutor? Would students be able to motivate and regulate themselves better? Would parents and administrators be able to gauge the overall progress of individual learners towards targeted learning outcomes? These are some of the questions addressed in this research under the scope of learning analytics.

Key words: mathematics experiences, learning analytics, data collection

1. INTRODUCTION – WHY IS IT SO HARD TO SUCCEED IN MATH?

Mathematics is a foundational subject in education. “Mathematics is often challenging for students with and without disabilities to master” (Little, 2009). It is a challenging subject to learn for many students because of four main reasons outlined below.

1.1 Building on previous knowledge

Difficulties in mathematics build up faster than any other subject since mathematics constantly challenges students to make statements and assumptions. Boaler (2013) indicates that new synapses are formed in the brain every time a mistake is committed by a student in mathematics and when the student thinks about why something is wrong. The reasoning process in mathematics is extremely intense, and if not well supported, will soon lead to a rapid fall in interest and capacity. “Math, more than most subjects, is based on sequential learning, and if students have missed any previous concepts, then they are unable to understand new ones” (Gregory, n.d.).

Learning mathematics is similar to building a tower. Each stone has to be securely placed for the tower to be unswerving. When a student studies a new mathematical concept, each of its pre-requisite concepts, co-requisite concepts, and other related concepts have to be well understood and reasonably mastered through diversified explanations and exercises. Further, the relevancy of the concepts and the seamless application of the concepts should also be mastered before the student feels at ease about mathematical problem-solving. Joseph’s (2009) study “shows that students must possess relevant knowledge and be able to coordinate their use of appropriate skills to solve problems.”

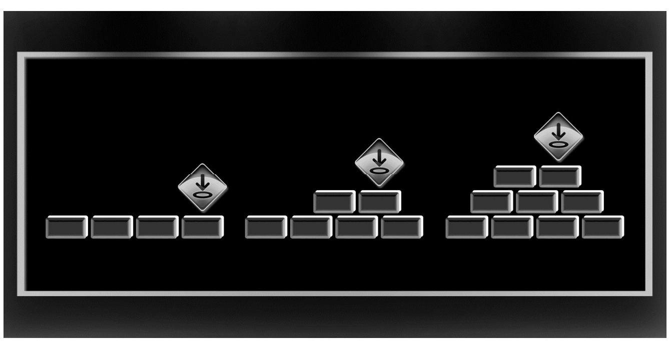

Figure 1- Sequential Learning Tower in Mathematics

1.2 Small procedural errors can lead to a destructive loop related to a loss of confidence

Chinn (2012) states that "many procedures that are taught for mathematics are very unforgiving on faulty memories. Often, even one small error in the application of a procedure is enough to generate failure." Even a small detail could have a substantial impact in mathematics, particularly when students who possess poor conceptual skills continue to solve an exercise using a rather wrong pathway without appropriate feedback. In such an incorrect state, students tend to make more erroneous personalized knowledge construction. Thus, it is important to address such wrong pathways, at real time, before they significantly affect the confidence of the student.

1.3 Confidence and performance are in a symbiotic relationship

Maher (1999) postulates that subject matter understanding emerges when new ideas fit into a larger framework of previously-assembled ideas. A metaphor that reflects this quite well is the notion that one assembles ideas in one's mind much as one assembles a jig-saw puzzle. Each new candidate piece, like each new idea, can be used only if it fits into the aggregate of pieces that have previously been assembled. Similarly, being constantly challenged to make decisions about what is correct and what is not, as a result of the assumptions the student has made while studying, the student may be confronted by some of the solutions and the process that led to the solutions that contradict some of the beliefs, thus affecting confidence. "The implication is that teachers and mathematics educators should focus on the possible difficulties faced by the students as they interact with the mathematical problem and problem solution" (Joseph, 2009). The support from teachers during and after a problem-solving process is crucial because one of the major factors that explains differences in self-confidence between children is their estimated competence (Nunes, Bryant, Sylva, & Barros, 2009). Estimated competence is often misjudged and students "routinely regard mistakes as indicators of their own low ability" (Boaler, 2013).

1.4 Need to support a student in real-time

Mathematical competence building is based on a personalized knowledge construction process. The ability to guide the learner to align, adjust, and consolidate the built competence is known as competence-based learning.

Maher (1999) indicates that a "greater recognition of the many thinking processes that must take place when anyone attempts to deal with a mathematical problem" should be part of the classroom practice. "Teachers, by listening to students, can have close contact with the ideas students are building in their minds. In this way, teachers can try to guide in the construction of those ideas" (Maher, 1999). Contemporary classrooms do not equip teachers to have access to ideas that students build in their minds as they solve problems.

During this mathematical knowledge construction process, the student will be challenged to build a highly personalized understanding of each mathematical problem-solving process. Inevitably, as part of the learning, the student will face facts that contradict the personalized knowledge being built. Maher affirms that "the student who is learning mathematics is supposed to build up a collection of ideas in his or her mind, but, in doing so, encounters fundamental ambiguities" (Maher, 1999). If these contradictions are not dealt with by the student or not detected and addressed by the teacher, they would weaken and erode the student's confidence over time. When these contradictions reach a threshold, a significant portion of confidence in mathematical problem-solving tends to collapse and the student would start to doubt the very construction process as well as the validity of the remaining personalized mathematical knowledge.

In general, only a particular set of points have to be aligned, adjusted, and consolidated in the personalized knowledge to retain the integrity of the entire knowledge structure. Stecker and Fuchs (2000) have shown that "student performance increased when teachers made instructional adjustments based on individualized curriculum-based measurement data." They conclude that "frequent assessment and linked instructional interventions are essential to increasing student mathematics performance."

It has been demonstrated by Parsons that the student performance plays a key role in building self-confidence because "the most important source of self-efficacy was found to be students' past experience of success or failure" and "it is argued that lecturers and support tutors might do more to develop students' confidence" (Parsons, Croft, & Harrison, 2011).

Competence-based learning (Levy & Ramim, 2015; Martinez, Avalos, Lopez, & Palacios, 2015) allows one to measure, predict, and address competences at various levels of granularity. It is important to observe and guide the evolution of competences during the process of their development. That is, as Joseph (2009) points out, students need to problematize their own learning, as and when learning happens.

In general, competence-based learning facilitates continuous and close monitoring of students' study and problem-solving activities to ensure that they learn positively and effectively. In today's classroom, it is impossible to give extensive individual attention to every student given the number of students in each class as well as the amount of time available during school day. In contemporary instruction, competence can only be assessed when students are assessed. Even if there could be one teacher per student during class, it is near impossible for the teacher to know about the student's homework habits and problem-solving processes. To be a perfect tutor, one would have to be, consistently if not always, available, seeing students' work, answering, and ready to provide useful hints, motivating instructions, and appropriate guidance. Clearly, this is impossible for a human teacher to offer, given contemporary instructional setup! Students are therefore left alone in their learning process and are taught from a distance even in a classroom situation.

This lack of close real-time support is the likely reason many capable students lose interest in mathematics.

2. CHALLENGES FOR TEACHERS

To know what the student knows is a key challenge for mathematics teachers. A more compelling challenge is to receive this piece of knowledge at the right time. An even bigger challenge is to obtain the study context within which the student is trying to acquire the knowledge. This study context relates to contextual elements surrounding the student as he or she is studying. For example, a student who is studying new mathematical concepts while being anxious because of family circumstances or health issues is clearly in a quality-altered study context. Finally, the most critical challenge is to positively engage the student in a pedagogically optimal manner to help gain targeted competences in mathematics. Teaching, explaining, gesticulating, animating, and overwhelming the student with instruction without this positive engagement would only yield a marginal improvement in the student's learning process and competences. In other words, learning has to be fully bidirectional: teachers giving full information to the students and students giving full information to the teachers. The notion of "fullness" indicates that in addition to information on teaching, learning and assessments that teachers and students share, one could also share information on alternative teaching methods for a group of students in the current learning context, learning techniques being followed by a student, social contexts of learning appropriate for a domain, personalized instructional mechanisms, and so on. In general, in current learning contexts, students are not able to give additional information about their learning, in addition to what can be gleaned from assessments, to teachers.

The greatest quality of a mathematics teacher is to have a good sense of students' understanding, to assist in the building and mastering of concepts, and to groom the personalized knowledge and logical thinking process while addressing the gaps in knowledge and process. This is what is referred to as "Listening to students' mathematical thinking" (Suurtamm & Vézina, 2010). How could a teacher possibly know what is going on in the head of each student? By marking the homework? Unfortunately, in contemporary education, students typically go through problem-solving processes and arrive at impasses or solutions to exercises, without sharing the processes that lead to impasses or solutions with teachers. These hidden efforts of the student and the unaccounted challenges the student faces are the root causes of confusion, discouragement, and failure among students.

Researchers have come to the conclusion that "teachers have an important role in guiding students' mathematical development by engaging them in problems, facilitating the sharing of their solutions, observing and listening carefully to their ideas and explanations, and discerning and making explicit the mathematical ideas presented in the solutions [Ball, 1993; Lampert, 2001; NCTM, 1991; NCTM, 2000]. Several research projects (see for example, Cobb, Wood, & Yackel, 1992; Fennema et al., 1996; Franke & Kazemi, 2001; Simon & Schifter, 1991) have found significant benefits when teachers attend to their students' mathematical thinking. The benefits included higher levels of conceptual understanding by students and more positive attitudes held by both teachers and students towards mathematics" (Suurtamm & Vézina, 2010).

Here is a typical example. A student begins his homework of multiplying polynomials. At first, he is puzzled and does not remember even what a polynomial is. After searching the internet and then going through the textbook, he reads an explanation that helps him to start the homework. But the material he has read has also raised more questions in his understanding. Is there always a coefficient in front of a polynomial? Is it possible that a polynomial has no variables? Are addition and subtraction operations included in a polynomial? In spite of these questions, the student begins to work on a polynomial multiplication problem. As he begins, he is still confused about the operation that must be completed first – is it the multiplication or the addition. He gives it a try and solves the exercise by doing the addition first followed by the multiplication operation, but the result seems to make no sense for him. He then tries again with a different sequence, again the result seems odd. He doesn't remember if exponents in likely terms must be added or multiplied when multiplying the terms. Finally, he decides to compute his answer with an online mathematics application on the internet. The online application offers a solution. After writing down the answer, he finds that his current answers are quite different from this solution. He is not sure how else the solution can be arrived. He stops working on the problem and postpones it for another time. The negative thoughts are already haunting him, making him suspect his mathematical abilities compared to his classmates. The next day, a smiling teacher works out the solution for the class and asks if anyone has any doubts. The student decides not to expose himself to the mockery of his classmates and does not reveal his doubts or the negative thoughts. He convinces himself that he will probably understand the polynomials concept down the road.

This example displays the ‘one way’ interaction where the student has not been able to give information to his teacher thus preventing this positive engagement needed for success. As outlined in this example, the current approaches to classroom mathematics education do not provide the teacher with critical information that is needed to understand the learning and problem-solving processes of a student. Without knowing it, the teacher is not in a position to engage the student in a deeper learning process.

One can imagine how the scenario would have been different if the teacher had followed the entire problem-solving process of the student during the evening and having chatted in real time with the student in private or even having a personal talk with him in the next morning before the start of the class about the issues he faced. Capture of the entire problem-solving process could be a gold mine for a teacher who wants to ‘hear’ about the knowledge of a student. These ‘captured’ exercises could potentially identify the weakness in the student’s understanding and application of mathematical concepts.

The sequence of solution steps submitted by the student and the final answer of the student only reveal a small portion of the student’s skills and knowledge gaps. Teachers need to have access to the ’captured’ problem-solving habits and challenges of the students. Students’ habits and challenges offer a way for the teacher to pinpoint concepts that need to be reinforced and mastered before going any further.

Mathematical weaknesses have the tendency to be covered up if the whole process of problem-solving is not ‘visible’ or ‘audible’. This is why it is necessary for an instructor to have a deeper and timely understanding of students’ progress in order to spot weaknesses as and when they occur.

The next biggest challenge for a teacher is to properly address the weaknesses related to mathematical concepts among his students at the right moment with appropriate evidences as to the necessity of such an address. The teacher needs to provide timely and personalized support according to each students’ individualized needs and difficulties.

With the current structure of the education system, it is extremely difficult for teachers to respond appropriately to every difficulty encountered by students. More often than not, teachers are unaware of the existing difficulties among their students.

3. CHALLENGES FOR STUDENTS

As mentioned before, mathematics constantly challenges the mind. While studying mathematics, each student is filled with questions, assumptions, hypothesis, conclusions, and statements. Each student also exerts a certain level of understanding and the ability to apply that understanding in problem-solving situations. Teachers need to ensure that students' comprehension is understood, grounded, and mastered. To optimize the teacher-student interaction, students need to have access to an open channel to share their questions, concerns, confusion, and successes as they study or work their math exercises. There are many crossroads during the process of problem-solving and students are expected to recollect and summarize past study experiences into the current problem solving context. The biggest challenge for students is to undergo the learning process with few inputs from their teachers and with few outlets to communicate the details that make them perplex and alter the validity of their answers.

It is very hard and discouraging for students to work on their own with no access to someone to whom they can ask for help or express difficulties, but rather having to rely on their own judgement and assumptions. Moreover, students who face difficulties in their work are not able to pinpoint the exact problem they faced. This leads to deterioration of their motivation and capability to share their difficulties with someone else. In general, students facing difficulty end up with a general statement such as 'I do not understand my math'.

Even students capable of logical thinking and mathematical reasoning may have under-expressed or suppressed misconceptions. Students may not have the capacity to recognize weaknesses and strengths. Students may emphasize more on their weaknesses than strengths, thus resulting in two significantly different viewpoints, a students' viewpoint and a teacher's viewpoint, as shown in Figure 2. While the teacher is able to discern the few concepts that need to be reinforced, the student often exaggerates the extent of a few misunderstandings into a generalized situation, and thus, based on a select set of bad results, concludes being incompetent in mathematical problem solving.

Figure 2 – A Student and Teacher's Different Viewpoints

Therefore, it is important that students express even little parts of problem-solving that confuse them as they work through a mathematical exercise. It would be a welcome change if students' problem-solving episodes along with specific difficulties that they faced are captured in an automated fashion, compiled, and presented to instructors through an application.

Such an advanced application, called ALEKS, is a Web-based system, working around "an artificial intelligence engine that assesses each student individually and continuously," "mapping the details of each student's knowledge." ALEKS is monitoring the learning process of each student and is able to provide the student with a "selection of only the topics" he is ready to learn at this exact moment. ALEKS also records successes and failures to guide the student for an optimal learning path through "one-on-one instruction, 24/7" (McGraw-Hill, 2015).

While ALEKS focusses on knowledge aspects, it does not provide an analysis of both the level of competence and the level of confidence of students which is crucial in learning mathematics.

4. MATHeX SOLUTION

MATHeX is a learning analytics tool that aims to study students' conceptual, problem-solving, and metacognitive behavior as they work through mathematical exercises. It is a companion software that is always there whenever students study mathematics and solve problems, 'observing' and 'listening' to their challenges, guiding them even in individual steps, giving them instructions, providing encouragement, and displaying their progress. The goal of MATHeX is positive engagement with students. The tool also focuses on finding ways to strengthen the study habits of students, creating opportunities for motivation and continuous informal assessment of their progress.

MATHeX features

MATHeX features can be summarized by these two key points: capturing data and analyzing data. These two points are explained in more detail below.

4.1 Capturing data

MATHeX aims to collect as much data as possible from the observations of students' mathematics related work. The tracking of data by MATHeX supplements the data inherently collected by the teacher, either in the form of listening to a student or classroom observations or assessment results. The collection of student's data is crucial in engaging the students in their learning process thus enabling and enhancing bidirectional communication between the teacher and the students.

The current version of MATHeX captures the following datasets:

4.1.1 Timestamps

The time dataset provides information on each study session (including reading and preparing to solve mathematical problems), the amount of time spent on each step of a problem-solving process, and the overall duration in each problem.

MATHeX also tracks the time of inactivity within a problem-solving process. After a preset time of inactivity, the software will invite the student to indicate the reason for inactivity from a list of options. The options include a time of absence from the study session, a time of reflection about the mathematical problem, a time of 'silence' created by confusion, and a time of being stuck needing help. This feature will also help estimate the attention span and concentration skills of students.

4.1.2 Pointer, mouse, menu, and keyboard activities

As the student works through exercises or assignment problems on a computer, including mobile devices, MATHeX tracks pointer, mouse, menu, and keyboard status of the devices. For example, mouse click statii during a problem-solving activity is captured and associated with the corresponding problem the student is attempting to solve. These datasets include the erasing of an answer, the reviewing of the problem statement, the changes made within a solution step, the help-seeking behaviour, and so on. This capacity to capture all writing, typing, and graphical activities on any computer with the standard internet browsing capacity is key to 'listening' to the student's mind during problem-solving processes. MATHeX aims to discover patterns in the solving process of each student, to have a better understanding of weaknesses, hesitations or skill achievements. In capturing the expressed (written, moused, typed, or selected) steps of a given problem, MATHeX is able to detect the confidence and the competence of the students in solving a particular problem type.

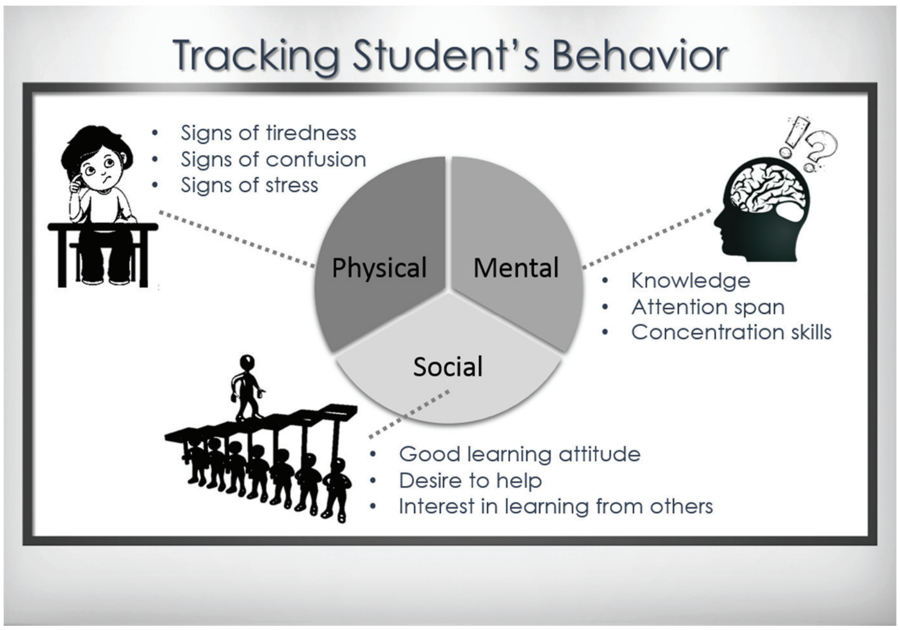

Figure 3 – MATHeX Tutoring Environment

MATHeX offers multiple learning environments such as individual, tutoring, social interactive, and 3D virtual learning spaces. Activities within each of these spaces are sensed and used to recognize problem-solving patterns.

4.1.3 Contextual information

The context of mathematical problem-solving experiences in MATHeX includes information such as the types of problems attended by the student, topic space covered by each problem, the study patterns of the student, the goals of the teacher, the help-seeking behaviour of the student, and the student's interactions with the problem-solving environment. In future work, MATHeX aims to capture visual data on students' eye movement and facial expression, as further validation of signs of confusion, stress or tiredness. Information that are not available through mouse and keyboard such as the heartrate and other physiological data can be attached to the context from wearable devices.

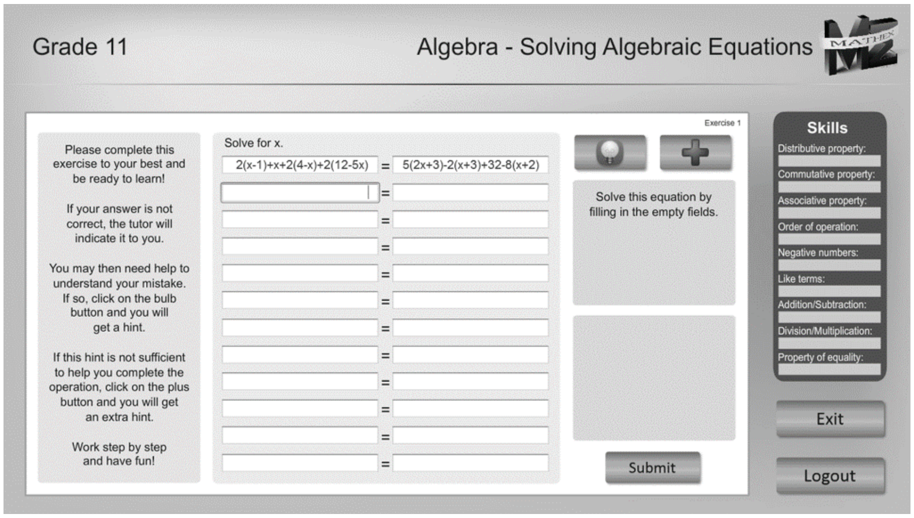

Figure 4 – Student's Behavior Data

MATHeX also aims to capture the social behaviour of students while engaged in problem-solving activities. Metacognitive traits such as self-regulation and co-regulation can be observed from within MATHeX with assistance from built-in tools such as Self- and Co-Regulated Learning (Zheng, 2015).

4.2 Analyzing data

Captured data are subjected to continuous analysis in MATHeX involving data transformation, data visualization, and regulation.

4.2.1 Transforming data

MATHeX uses an analysis engine named SCALE (Boulanger, 2015) that transforms raw data in meaningful data. The analysis results in an assessment of students' competence and confidence based on the observed mathematical problem-solving experiences.

The analysis engine (SCALE) receives structured data in a MathML content format and will then analyze the validity of the answer and the overall problem-solving process of the student.

MATHeX aims to measure weaknesses identified in a student's understanding in terms of targeted skills. Observations on the speed and accuracy of the problem-solving process, in conjunction with the correctness of the solution, can be mapped to individual problem-solving skills. Such an individualized observational process guides the student through needed supplemental exercises and activities. The growth of individual skills can be transformed into competences using a competence framework such as the Bloom's Taxonomy (Maker & Nielson, 1995; Highley & Edlin, 2009).

In the tutoring environment, a competence-based analysis will assist in the offering of real-time feedback to the student.

4.2.2 Visualizing data

Data can be valuable if they are visually accessible and meaningful. MATHeX offers displays of pertinent information for students, teachers, parents, and school administrators in an interactive dashboard. This visualization is key to engage students and motivate them to reflect on their performances throughout the learning process. MATHeX dashboard also provides visuals on the competences of the student. MATHeX's teacher dashboard enables an overall picture of the entire class. The dashboard will also enable teachers to communicate with students at real-time to offer feedback on specific competences and struggles experienced by students.

The dashboard will help parents to see ranking of a student in comparison with the rest of the class.

The student's view of the dashboard provides information that associates a student's problem-solving performances with the expected outcomes.

4.2.3 Self- and co-regulation

MATHeX includes a self-regulation feature that allows students to create their own initiatives, in a tool called SCRL (Zheng, 2015).

Allowing students to regulate their own study behaviour engages students in their activities as no teachers, as motivated as they can be, would be able to do. MATHeX encourages student's self-regulation knowing that "undoubtedly, all learners are responsive to some degree during instruction; however, students who display initiative, intrinsic motivation and personal responsibility achieve particular academic success" (Zimmerman, 1990).

Students will have the opportunity to motivate and regulate themselves by interacting with their competence levels in the dashboard and by creating their new initiatives, setting new goals, picking out strategies to achieve these goals, and actually achieving these goals. Students will be accompanied by

MATHeX till the completion of their initiatives and will be guided for better success.

MATHeX also aims to provide a co-regulation aspect that would encourage peers to help each other and assist one another with respect to individual initiatives. This aspect will also create new datasets as to the social engagement of students with their classmates. DiDonato's (2006) study reveals that when students help each other, they learn tremendously and they are encouraged to develop socially. Similar results have been noted in the peer tutoring research (Kumar, 1996). Consistent with the old adage: 'to teach is to learn twice,' research on peer tutoring has found increases in both the tutee's and tutor's academic and social development as a result of peer tutoring interventions (DiDonato, 2006). "In these cases, there are a number of academic and social benefits to all group members as a result of participating in co-regulatory processes." That is, co-regulation may have benefits for both the person doing the regulating and other group members to whom the action is directed, and this may lead to increases or refinements in both students' SRL" (DiDonato, 2006).

Moreover, Perger (2013) also came to the conclusion that "working with others was another practice recognized by both adult experts and students as important when learning mathematics. Teachers need to develop learning environments and practices that encourage students to work in groups. The teacher works as a co-ordinator providing guidance and support both in mathematics content learning and in developing skills that enable students to work together. This ability to work together has been recognised as a skill students need to be taught" (Perger, 2013).

Mathematics is a complex subject and at the same time can be explained in simple terms. Experiencing the same concept in different contexts might enlighten a confused student. With this perspective in mind, Boaler (2013) expresses how "encouragement of a growth mindset culture will require schools to move to grouping practices that do not label or send negative messages to students, and teaching approaches that value the thinking, struggles and varied learning pathways of all students."

5. CONCLUSION

Many online applications allow students to learn mathematical concepts and train themselves. Among those is ALEKS, which focusses on the knowledge aspects (McGraw-Hill, 2015). However, this research aims at an analytics tool that would provide more than just an application to learn and be trained. Such an analytics tool allows students to get precise feedback about what they master and what they don't, to be guided through specific exercises that respond to their individual needs, and to enable teachers to know exactly about the problem-solving process that the students have experienced.

MATHeX aims to provide analysis of both the level of competence and the level of confidence of students. Further, MATHeX looks to capture study habits, social contexts, and inclination of students. In MATHeX, graphs in the dashboard are interactive and allows the students to create their own initiatives through the self- and co-regulated embedded learning system. Moreover, MATHeX enhances mathematics study experiences by allowing students to work any types of problems and capturing their work to analyze both strengths and weaknesses during the solving process. In addition to ALEKS' features, MATHeX helps the teachers understand the problem-solving process undertaken by the students, and know where the students struggled, and what are their competence, confidence, and metacognitive abilities. MATHeX is a tool intended to 'listen' to the students' mind during problem-solving processes thus providing this open channel that will allow the teacher to understand what the students would like to say and engage them positively. In capturing the expressed (written, moused, typed, or selected) steps of a given problem, MATHeX is able to detect the confidence and the competence of the students in solving a particular problem type.

It is important to note that the observed and inferred data from students' interactions can only be shared with others, including teachers, parents, and administrators, with the expressed consent of the student. The data is inherently owned by the student and the student's permission is explicitly sought to use the data for analytics and share the data with others.

Building strong mathematics skills needs tutor involvement, passion, care, time, and devotion. The key interest of a mathematics teacher is to know how the student has solved his problem, what difficulties he faced, how much time he took to be successful, and much more. In today's classroom, it is impossible to give this individual attention to every student. MATHeX wants to bring solutions by being a learning analytics tool that accompanies the teacher and the student, captures the student's overall activities, identifies weaknesses through analysis, guides the student and provides instruction, gives feedback, and displays visually all needed information in a dashboard. The purpose of this study is to know if learning analytics would enhance the mathematics experiences of the student and to discover the role of learning analytics in mathematics education. It is also the goal of this research to learn about the best approach in classroom between the paper and the computer. These approaches will be evaluated using the results of the students and their feelings towards it.

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