THE ARITHMETIC OF FRACTIONS

- multiplication, division
- addition, subtraction

What to do and what *not* to do.

THE RATIONAL NUMBERS

and their operations

REVIEW:

Recall that a **rational number** is simply a *well defined* fraction, meaning one having a *non-zero* denominator.

Moreover, two or more fractions may have the same numerical value even though their numerators and denominators are not identical.

For example, 4/8 and 5/10 are both equal to ½, but their numerators and denominators do not match.

In this case, they are said to be **equivalent**.

The Fundamental Law of Fractions

The value of a fraction is NOT changed if BOTH numerator and denominator are either multiplied or divided by the same non-zero number.

THE RATIONAL NUMBERS

and their operations

The rational number system has five arithmetical operations.

Four of them - multiplication, division, addition, and subtraction - we will study in detail in this tutorial.

The fifth, that of exponentiation (a special case of multiplication), will be covered as a topic in its own right.

Division and Subtraction of Rational Numbers as Special Cases of Multiplication and Addition

We are going to discover that division of fractions is simply a special case of multiplication and that

subtraction of fractions, as in the case of whole numbers, may be interpreted as the addition of signed numbers.

Multiplying Fractions

What to do and what *not* to do

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Multiplying Rational Numbers What to do

The numerator of the product of two fractions is found by multiplying the numerators of each of them together.

The denominator of the product of two fractions is found by multiplying the denominators of each of them together.

$$\left(\frac{p}{q}\right)\cdot\left(\frac{m}{n}\right) = \frac{p\cdot m}{q\cdot n}$$ where p, m, q, n are integers.

$$q, n \neq 0$$

EXAMPLE 1

$$\left(\frac{4}{7}\right)\cdot\left(\frac{-1}{3}\right) = \frac{4\cdot(-1)}{7\cdot 3} = \frac{-4}{21} = -\frac{4}{21}$$

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Multiplying Rational Numbers

$$\left(\frac{p}{q}\right)\cdot\left(\frac{m}{n}\right) = \frac{p\cdot m}{q\cdot n}$$

where p, m, q, n are integers.

$q, n \neq 0$

The Fundamental Law of Fractions

The value of a fraction is NOT changed if BOTH numerator and denominator are either multiplied or divided by the same non-zero number.

EXAMPLE 2 In this case, BOTH numerator and denominator are divided first by 5 and then by 4.

$$\left(\frac{5}{-8}\right)\cdot\left(\frac{4}{-5}\right) = \frac{5\cdot 4}{(-8)\cdot(-5)} = \frac{\overset{1}{\cancel{5}}\cdot\overset{1}{\cancel{4}}}{\underset{(-2)}{\cancel{(-8)}}\cdot\underset{(-1)}{\cancel{(-5)}}}$$

This technique of simplifying fractions is called **cancelling common factors** across the fraction line.

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Multiplying Rational Numbers

$$\left(\frac{p}{q}\right)\cdot\left(\frac{m}{n}\right) = \frac{p\cdot m}{q\cdot n}$$

where p, m, q, n are integers.

$q, n \neq 0$

The Fundamental Law of Fractions

The value of a fraction is NOT changed if BOTH numerator and denominator are either multiplied or divided by the same non-zero number.

EXAMPLE 2 continued

$$\left(\frac{5}{-8}\right)\cdot\left(\frac{4}{-5}\right) = \frac{5\cdot 4}{(-8)\cdot(-5)} = \frac{\cancel{5}^{1}\cdot\cancel{4}^{1}}{(-\cancel{8}^{(-2)})\cdot(-\cancel{5}^{(-1)})}$$

$$= \frac{1\cdot 1}{(-2)\cdot(-1)} = \frac{1}{2}$$

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Multiplying Rational Numbers

$$\left(\frac{p}{q}\right)\cdot\left(\frac{m}{n}\right)\cdot\left(\frac{r}{s}\right) = \frac{p\cdot m\cdot r}{q\cdot n\cdot s}$$

where p, m, r, q, n, s are integers.
$q, n, s \neq 0$

EXAMPLE 3

In the case of multiplying three or more fractions, simply multiply through the numerators and multiply through the denominators as shown by the rule above.

$$\left(\frac{1}{4}\right)\cdot\left(\frac{-3}{2}\right)\cdot\left(\frac{-3}{2}\right) = \frac{1\cdot(-3)\cdot(-3)}{4\cdot 2\cdot 2} = \frac{9}{16}$$

In this example, there is **no cancellation of common factors** across the fraction line because there are no factors which occur both in the numerator and the denominator.

Dividing Fractions

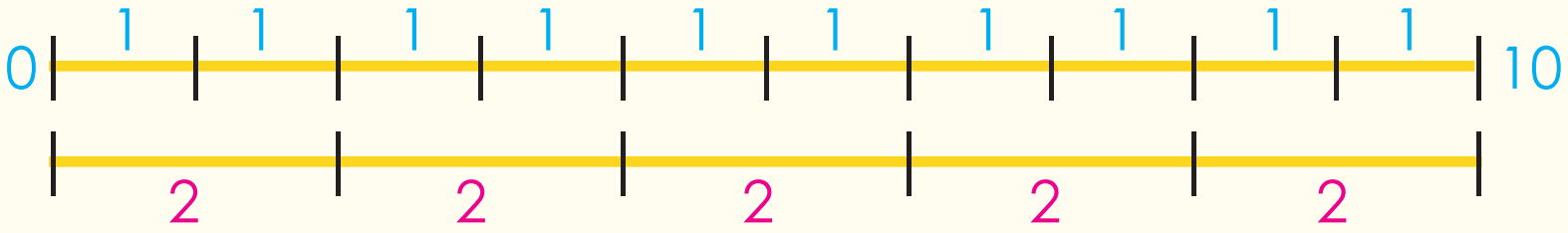
What to do and what *not* to do

THE RATIONAL NUMBERS
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Dividing Rational Numbers

Let's think a little bit about the division of two numbers.

To divide $\frac{10}{2}$, we are asking the number of times 2 fits into 10.



ANSWER: $\frac{10}{2} = 5$

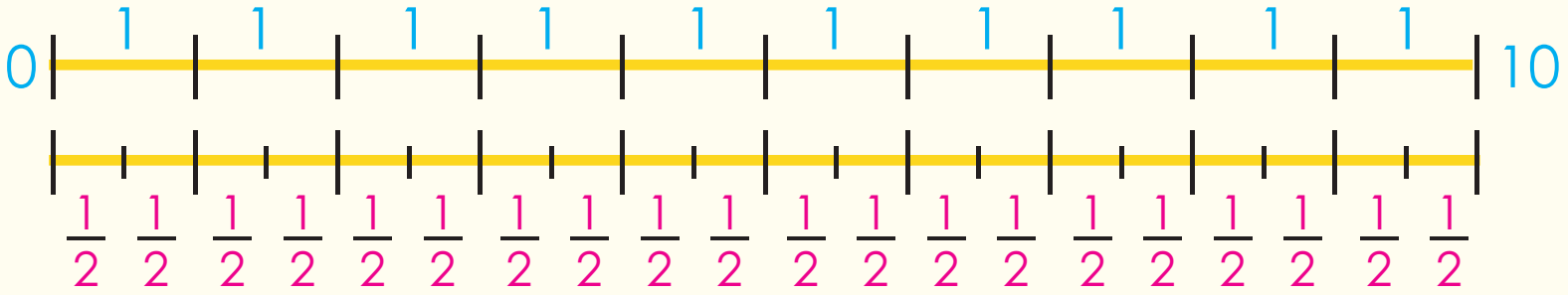
because the number of times 2 fits into 10 is 5 times.

THE RATIONAL NUMBERS
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Dividing Rational Numbers

Now consider the following division.

To divide $\frac{10}{\frac{1}{2}}$, we are asking the number of times $\frac{1}{2}$ fits into 10.



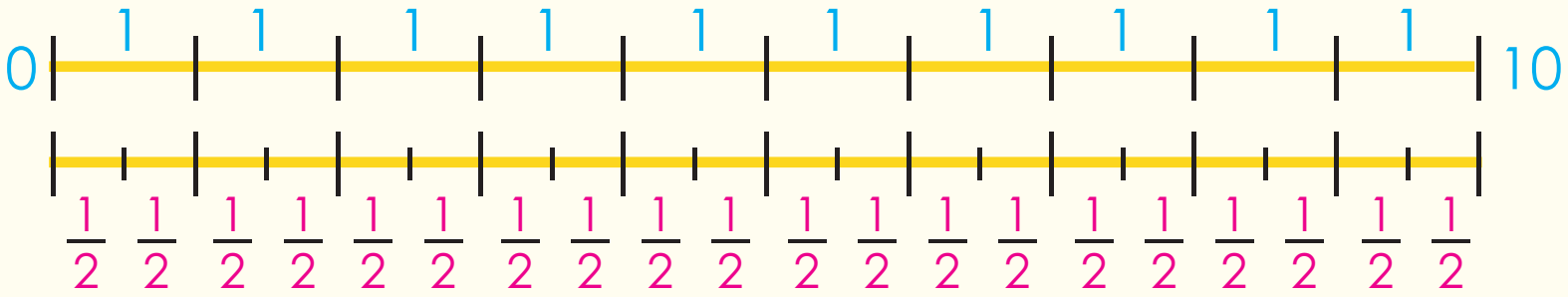
ANSWER: $\frac{10}{\frac{1}{2}} = 20$

because the number of times $\frac{1}{2}$ fits into 10 is 20 times.

THE RATIONAL NUMBERS
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Dividing Rational Numbers

Let's think a bit about what dividing by a fraction really means.



Because $\frac{1}{2}$ fits into 1 **twice**, we can say that $\frac{1}{\frac{1}{2}} = 2$.

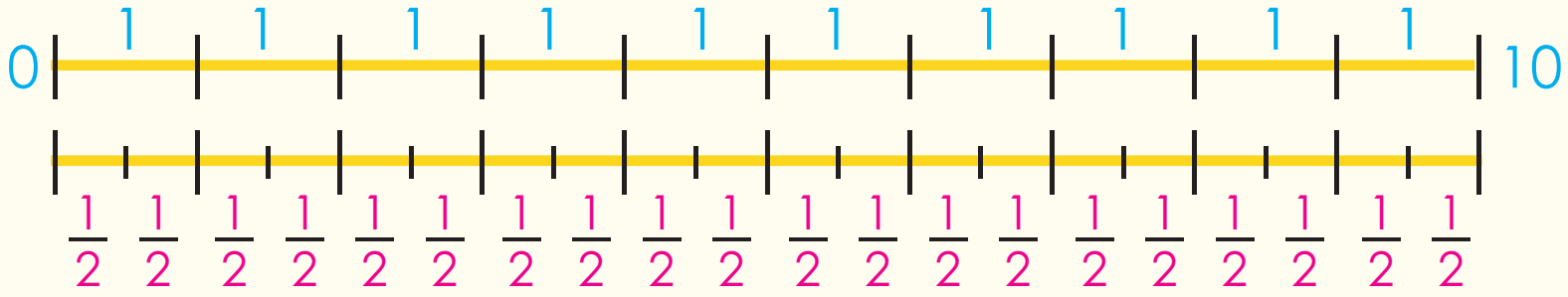
Therefore, $\frac{1}{2}$ will fit into 4 **eight** times; so $\frac{4}{\frac{1}{2}} = 8$.

Moreover, we already know that $\frac{10}{\frac{1}{2}} = 20$.

It seems that **dividing** a number by $\frac{1}{2}$ is simply
a process of doubling the numerator, or **multiplying** it by 2 .

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Dividing Rational Numbers



From our previous discussion,
it appears that **dividing** a number by $\frac{1}{2}$ is simply
a process of doubling the numerator, or **multiplying** it by 2 .

The next thing to notice is that $2 = \frac{2}{1}$ is the reciprocal of $\frac{1}{2}$.

From there is not difficult to deduce, from the diagram above, that

dividing a number by $\frac{1}{4}$ is simply a process of quadrupling
the numerator, or **multiplying** it by 4 , which is the **reciprocal** of $\frac{1}{4}$.

The rule for the division of fractions follows from these observations.

THE RATIONAL NUMBERS
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Dividing Rational Numbers

What to do

Division of two fractions is simply the **multiplication** of the numerator fraction by the *reciprocal* of the denominator fraction.

$$\left(\frac{p}{q}\right) \div \left(\frac{m}{n}\right) = \frac{\frac{p}{q}}{\frac{m}{n}}$$

where p, m, q, n are integers.

$q, n, m \neq 0$

$$= \left(\frac{p}{q}\right) \cdot \left(\frac{n}{m}\right) = \frac{p \cdot n}{q \cdot m}$$

EXAMPLE 1

$$\left(\frac{11}{-6}\right) \div \left(\frac{2}{5}\right) = \left(\frac{11}{-6}\right) \cdot \left(\frac{5}{2}\right)$$

$$= \frac{11 \cdot 5}{(-6) \cdot (2)} = \frac{55}{-12} = -\frac{55}{12}$$

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$$\left(\frac{p}{q}\right) \div \left(\frac{m}{n}\right) = \left(\frac{p}{q}\right) \cdot \left(\frac{n}{m}\right)$$

where p, m, q, n are integers.

$q, n, m \neq 0$

The Fundamental Law of Fractions

The value of a fraction is NOT changed if BOTH numerator and denominator are either multiplied or divided by the same non-zero number.

EXAMPLE 2 To simplify the answer, BOTH numerator and denominator are divided first by 3 and then by (-7).

$$\left(\frac{3}{-49}\right) \div \left(\frac{15}{-14}\right) = \frac{3 \cdot (-14)}{(-49) \cdot 15} = \frac{\cancel{3}^{1} \cdot (\cancel{-14}^{2})}{(\cancel{-49}^{7}) \cdot \cancel{15}^{5}} = \frac{2}{35}$$

$$\left(\frac{3}{-49}\right) \cdot \left(\frac{-14}{15}\right)$$

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Dividing Rational Numbers

Division of two fractions is simply the **multiplication** of the numerator fraction by the *reciprocal* of the denominator fraction.

EXAMPLE 3 What to do

In the case of two (or more) divisions, when brackets are present:

1. divisions within brackets are performed first.

$$\left(\left(\frac{3}{-49}\right) \div \left(\frac{15}{-14}\right)\right) \div \left(\frac{4}{9}\right) =$$

$$\left(\left(\frac{3}{-49}\right) \cdot \left(\frac{-14}{15}\right)\right) \div \left(\frac{4}{9}\right) = \left(\frac{\cancel{2}^{1}}{35}\right) \cdot \left(\frac{9}{\cancel{4}_{2}}\right) = \frac{9}{70}$$

$$\| \quad \frac{3 \cdot (-14)}{(-49) \cdot 15} = \frac{\cancel{3}^{1} \cdot (\cancel{-14}^{2})}{(\cancel{-49}_{7}) \cdot \cancel{15}^{5}} = \frac{2}{35}$$

THE RATIONAL NUMBERS
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Dividing Rational Numbers

Division of two fractions is simply the **multiplication** of the numerator fraction by the *reciprocal* of the denominator fraction.

EXAMPLE 4 What to do

In the case of two (or more) divisions, when brackets are present:

1. divisions within brackets are performed first.

$$\left(\frac{3}{-49}\right) \div \left(\left(\frac{15}{-14}\right) \div \left(\frac{4}{9}\right)\right) =$$

$$\left(\frac{3}{-49}\right) \div \left(\left(\frac{15}{-14}\right) \cdot \left(\frac{9}{4}\right)\right) = \left(\frac{\cancel{3}^{1}}{\cancel{-49}_{7}}\right) \cdot \left(\frac{\cancel{(-56)}^{8}}{\cancel{135}_{45}}\right) = \frac{8}{315}$$

$$\Downarrow$$

$$\frac{15 \cdot 9}{(-14) \cdot 4} = \frac{135}{(-56)}$$

THE RATIONAL NUMBERS
and their operations

Dividing Rational Numbers

Division of two fractions is simply the **multiplication** of the numerator fraction by the *reciprocal* of the denominator fraction.

EXAMPLE 5 What to do

In the case of two (or more) divisions with no bracketspresent:

1. perform the divisions in order from left to right.

$$\left(\frac{3}{-49}\right) \div \left(\frac{15}{-14}\right) \div \left(\frac{4}{9}\right) =$$

$$\left(\left(\frac{3}{-49}\right) \div \left(\frac{15}{-14}\right)\right) \div \left(\frac{4}{9}\right) =$$

$$\left(\left(\frac{3}{-49}\right) \cdot \left(\frac{-14}{15}\right)\right) \div \left(\frac{4}{9}\right) = \left(\frac{\cancel{2}^{1}}{35}\right) \cdot \left(\frac{9}{\cancel{4}_{2}}\right) = \frac{9}{70}$$

Adding Fractions

What to do and what *not* to do

THE RATIONAL NUMBERS
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Adding Rational Numbers

Let's think a bit about how we collect things. If we collect stamps, then in order to **add** to our collection, we must have another stamp - meaning an object of the same type.

This is also true in mathematics.
In order to add two numbers together they must be of similar 'type' or in some way 'like each other'.

This notion of 'likeness' is defined differently depending on the context.

For example, two rational numbers may be collected together, or **added together**, if they have the same denominators.

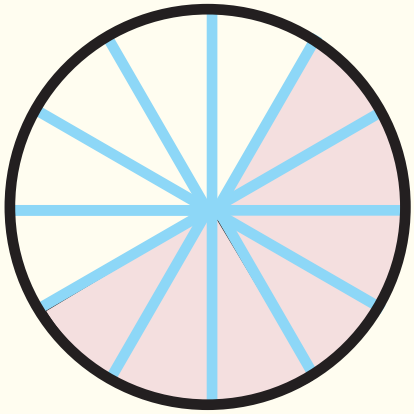
Why that is comes from the idea that a fraction is the relative magnitude of a part of a whole.
The denominator is indicative of the value of the whole.

THE RATIONAL NUMBERS
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Adding Rational Numbers

Let's consider, for a moment, the idea that a fraction is the relative magnitude of a part of a whole and that

the denominator is indicative of the value of the whole.



$\frac{7}{12}$

The fraction (the shaded area above) $\frac{7}{12}$ is the

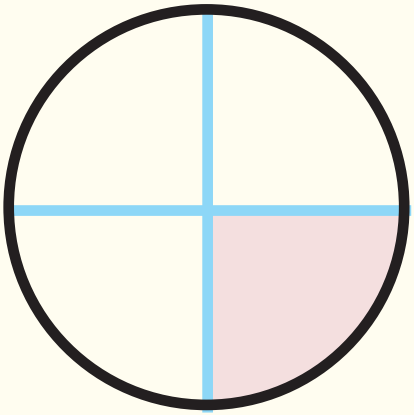
relative magnitude of the part, consisting of 7 pieces,
of a whole containing a total of 12 pieces of equal unit value.

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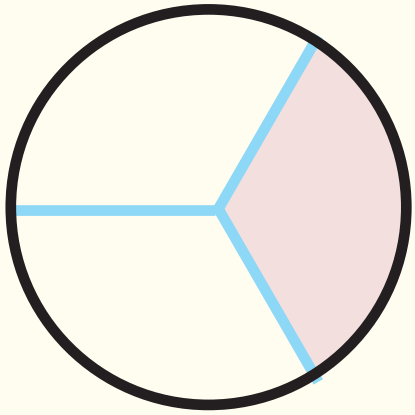
Adding Rational Numbers

To justify why two rational numbers may be collected together, or **added together**, only if they have the same denominators, we visualize a fraction geometrically as a piece of a pie which is divided into equal unit parts.

Cutting the pie into 3, 4, or even 7 pieces will determine the shape of each piece. Only similar size pieces can be collected together.



1 out of a total of 4 parts $\frac{1}{4}$



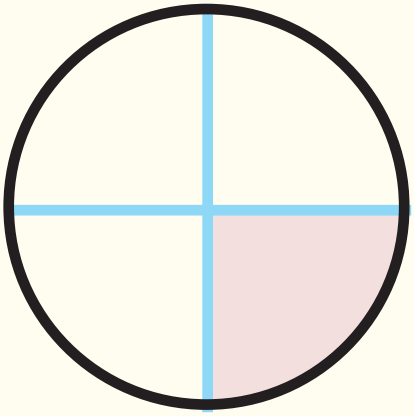
1 out of a total of 3 parts $\frac{1}{3}$

These fractions cannot be **added together** *unless* they have the same denominators because, geometrically, the fractional shapes are not of the 'same type'.

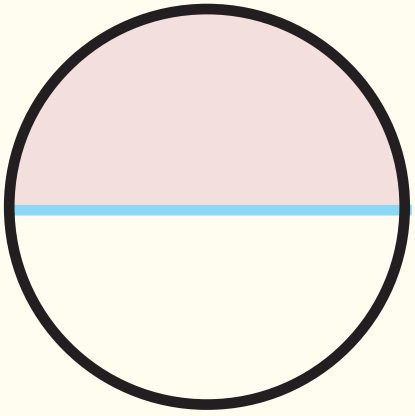
THE RATIONAL NUMBERS
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Adding Rational Numbers

Two rational numbers may be collected, or **added together**, if they have the same denominators.



1 out of a total of 4 parts $\frac{1}{4}$



1 out of a total of 2 parts $\frac{1}{2}$

Strictly speaking, these fractions may not be **added together** either, because they do not have the same denominators.

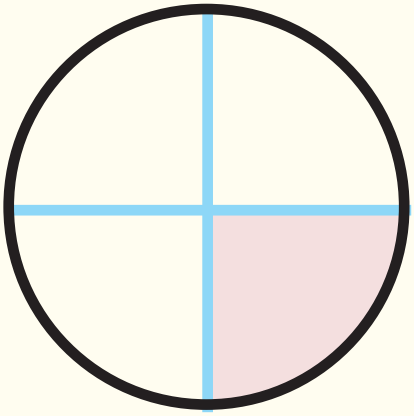
However, some of you may have remembered the **Fundamental Law of Fractions**.

Using it, the fraction $\frac{1}{2}$ is equivalent to the fraction $\frac{2}{4}$.

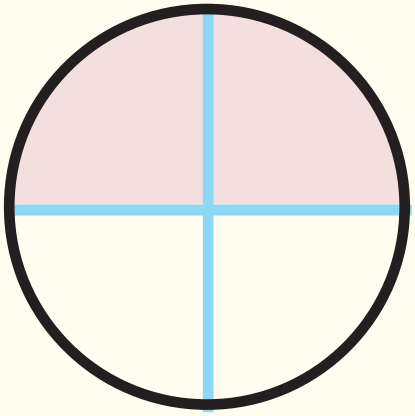
THE RATIONAL NUMBERS
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Adding Rational Numbers

Two rational numbers may be collected, or **added together**, if they have the same denominators.



1 out of a total of 4 parts $\frac{1}{4}$



1 out of a total of 2 parts $\frac{1}{2} = \frac{2}{4}$

These fractions now may be **added together** and collected into a single number (that is, may fit nicely into a single pie which is divided into quarters).

Simply **add the numerators** together and place it over the **common denominator**.

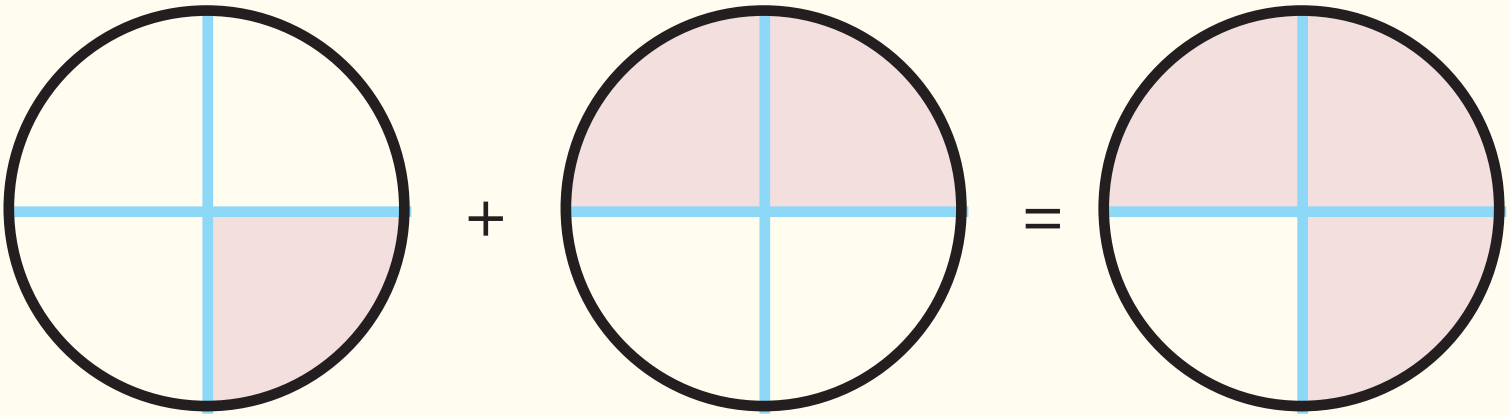
$$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$$

THE RATIONAL NUMBERS
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Adding Rational Numbers

Two rational numbers may be collected, or **added together**, if they have the same denominators.

Simply **add the numerators** together and place it over the **common denominator**.



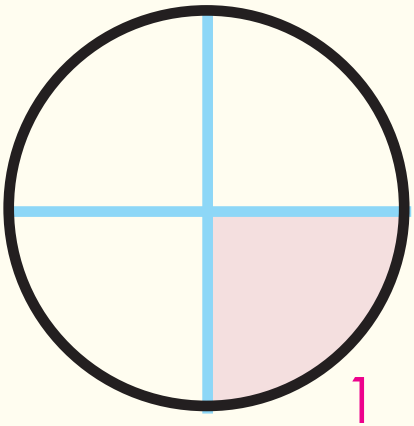
$\frac{1}{4}$ $\frac{2}{4}$ $\frac{3}{4}$

$$\frac{1}{4} + \frac{2}{4} = \frac{1+2}{4} = \frac{3}{4}$$

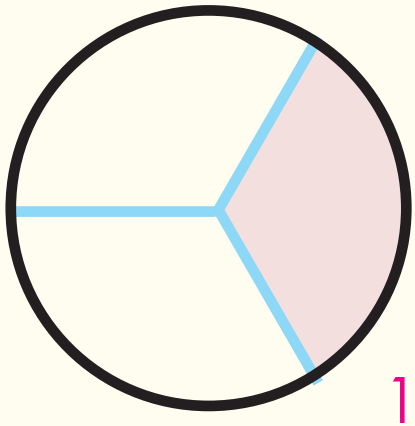
THE RATIONAL NUMBERS
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Adding Rational Numbers

Let's return to our previous example.



$\frac{1}{4}$



$\frac{1}{3}$

These fractions require a common denominator in order to be **added together**.

How can we obtain equivalent fractions for both of these which have the same denominator?

Use the **Fundamental Law of Fractions**.

$$\frac{1}{4} = \frac{1 \cdot 3}{4 \cdot 3} = \frac{3}{12} \qquad \frac{1}{3} = \frac{1 \cdot 4}{3 \cdot 4} = \frac{4}{12}$$

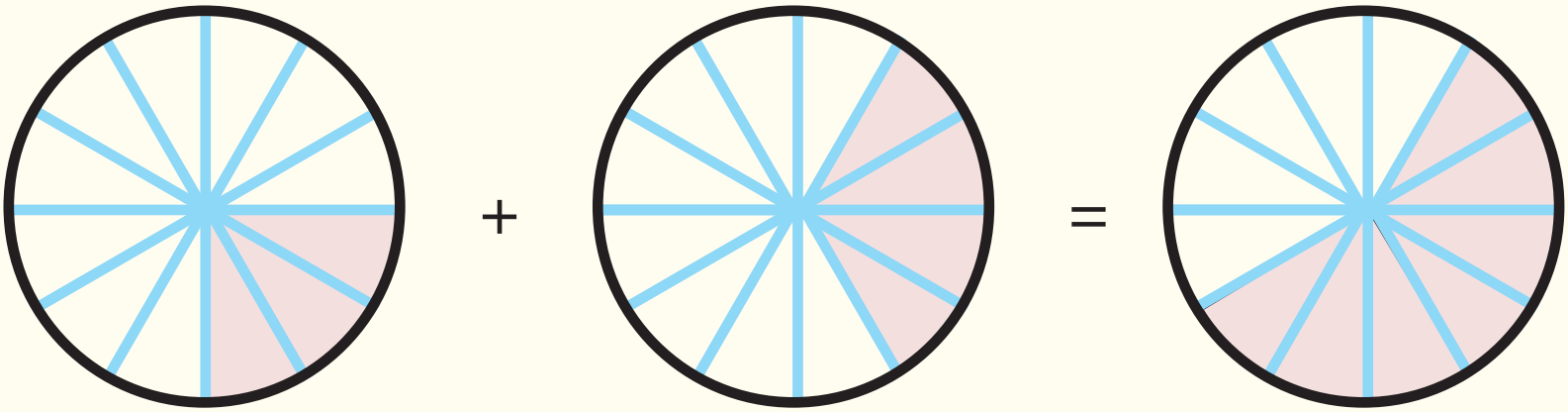
THE RATIONAL NUMBERS
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Adding Rational Numbers

A common denominator of $\frac{1}{4}$ and $\frac{1}{3}$ is 12.

Other common denominators of these two fractions exist but 12 is the smallest number containing both 4 and 3 .

To add the fractions together, simply **add the numerators** and place the sum over the **common denominator**.



$\frac{1}{4} = \frac{3}{12}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{7}{12}$

$$\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{3+4}{12} = \frac{7}{12}$$

THE RATIONAL NUMBERS
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Adding Rational Numbers What to do

$$\left(\frac{p}{q}\right) + \left(\frac{m}{n}\right) := \frac{p \cdot n + m \cdot q}{q \cdot n}$$

formally where p, m, q, n are integers.

$q, n \neq 0$

Step 1 is to find a common denominator for both fractons.
The smallest one, called the lowest common denominator (LCD), is the best choice - but not the only choice.

$q \cdot n$ is one possible choice for the above two fractions because both denominators q and n are present.

Recall that, to be able to add $\frac{1}{4} + \frac{1}{3}$, we needed a $4 \cdot 3 = 12$ piece pie.

Step 2 is to find equivalent fractions to the original ones, BOTH having the chosen common denominator.

$$\left(\frac{p}{q}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) \qquad \left(\frac{m}{n}\right) = \left(\frac{m \cdot q}{n \cdot q}\right)$$

NOTE $q \cdot n = n \cdot q$

THE RATIONAL NUMBERS
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Adding Rational Numbers What to do

$$\left(\frac{p}{q}\right) + \left(\frac{m}{n}\right) := \frac{p \cdot n + m \cdot q}{q \cdot n}$$

formally

p, m, q, n are integers.

$q \; n \neq 0$

Step 3 is to ADD the equivalent fractions together, by ADDING THE NUMERATORS and placing the sum over the chosen common denominator.

The equivalent fractions:

$$\left(\frac{p}{q}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) \qquad \left(\frac{m}{n}\right) = \left(\frac{m \cdot q}{n \cdot q}\right)$$

NOTE $q \cdot n = n \cdot q$

Performing the addition:

$$\left(\frac{p}{q}\right) + \left(\frac{m}{n}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) + \left(\frac{m \cdot q}{n \cdot q}\right) = \frac{p \cdot n + m \cdot q}{q \cdot n}$$

NOTE $q \cdot n = n \cdot q$

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$$\left(\frac{p}{q}\right) + \left(\frac{m}{n}\right) := \frac{p \cdot n + m \cdot q}{q \cdot n}$$

where p, m, q, n are integers.

$q, n \neq 0$

EXAMPLE $\left(\frac{5}{6}\right) + \left(\frac{-1}{9}\right)$ What to do

Step 1 is to find a common denominator. The smallest one, called the lowest common denominator (LCD), is the best choice - but not the only choice.

$6 \cdot 9$ is one possible choice for the above two fractions because both denominators 6 and 9 are present.

To choose $6 \cdot 9$ as the common denominator would be tantamount to dividing the pie into 54 pieces.

However, both 6 and 9 fit into a much smaller number than 54. In fact, both fit into (or divide into) 36. **Is this the smallest?**

EXAMPLE continued

What is the smallest number into which both 6 and 9 fit?

Finding LCD's

To find the lowest common denominator (LCD) from 6 and 9 we proceed as follows:

1. Factor both denominators into primes (i.e. until they can't be factored further). $6 = 2 \cdot 3$ $9 = 3 \cdot 3$

2. Collect all of the ***distinct*** factors in both denominators and take the ***maximum number of each*** appearing in either one.

distinct factors in both 2 , 3

maximum number in each: one factor of 2 **; two factors of** 3

3. Multiply together the ***maximum number of each*** **distinct factor** appearing in either denominator. This is the **LCD**.

LCD for $\frac{5}{6}$ and $\frac{-1}{9}$ is: $2 \cdot 3 \cdot 3 = 18$

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EXAMPLE continued

LCD for $\frac{5}{6}$ and $\frac{-1}{9}$ is: $2 \cdot 3 \cdot 3 = 18$

Step 2 is to find equivalent fractions to the original ones, BOTH having the chosen common denominator.

$$\left(\frac{5}{6}\right) = \left(\frac{5 \cdot 3}{6 \cdot 3}\right) \qquad \left(\frac{-1}{9}\right) = \left(\frac{(-1) \cdot 2}{9 \cdot 2}\right)$$

Step 3 is to ADD the equivalent fractions together, by ADDING THE NUMERATORS and placing the sum over the chosen common denominator.

$$\left(\frac{5}{6}\right) + \left(\frac{-1}{9}\right) = \left(\frac{5 \cdot 3}{6 \cdot 3}\right) + \left(\frac{(-1) \cdot 2}{9 \cdot 2}\right)$$

$$= \frac{15 + (-2)}{18} = \frac{13}{18}$$

THE RATIONAL NUMBERS
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Adding Rational Numbers

EXAMPLE $\left(\frac{5}{4}\right) + \left(\frac{-3}{5}\right)$ What *not* to do

$$\left(\frac{5}{4}\right) + \left(\frac{-3}{5}\right) \neq \left(\frac{\cancel{5}^{1}}{4}\right) + \left(\frac{-3}{\cancel{5}_{1}}\right)$$

DO NOT cancel common factors across a + sign.

$$\neq \frac{1 + (-3)}{4 + 1} = \frac{-2}{5}$$

DO NOT add numerators without a common denominator.

Subtracting Fractions

What to do and what *not* to do

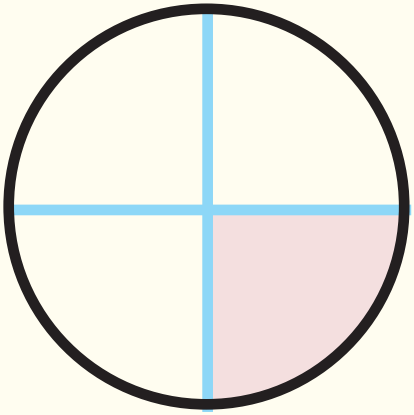
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Subracting Rational Numbers

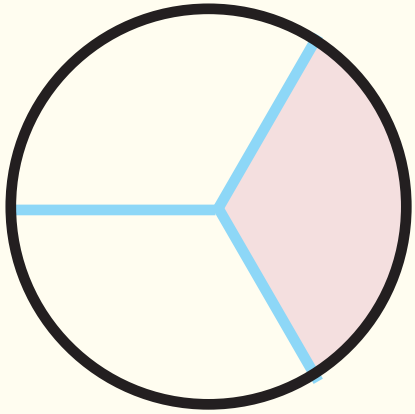
For the same reasons and in the same way that two rational numbers may be **added together**, only if they have the same denominators,

so two fractions may be **subtracted** from one another, only if they have the same denominators.

Only similar size unit pieces can be removed, or **subtracted**, from the whole or any part of it.



1 out of a total of 4 parts $\frac{1}{4}$



1 out of a total of 3 parts $\frac{1}{3}$

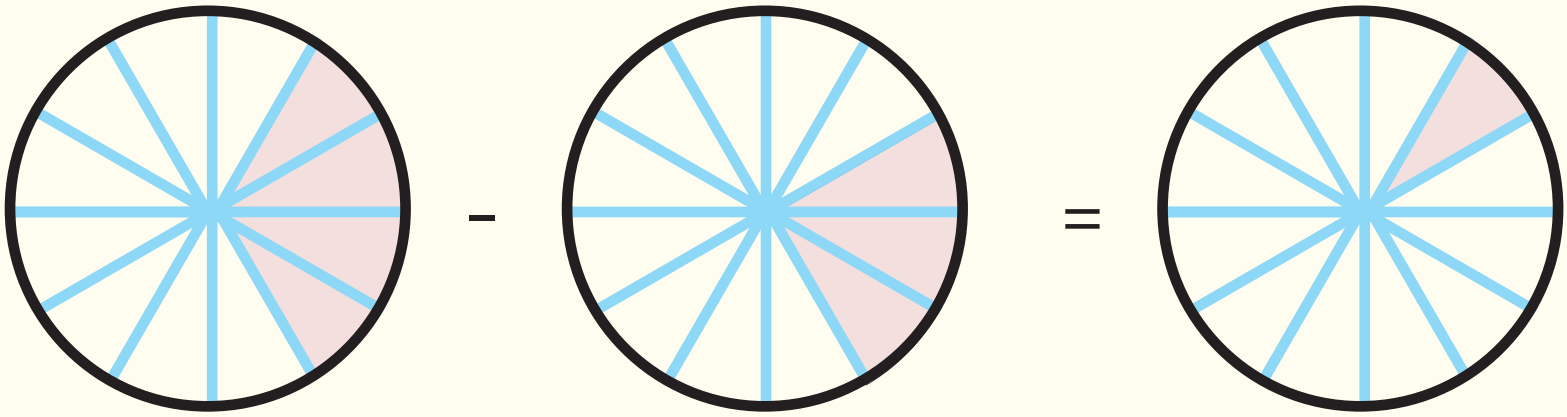
These fractions cannot be **subtracted from one another** *unless* they have the same denominators, because the fractional shapes are not of the 'same unit type'.

THE RATIONAL NUMBERS
and their operations

Subracting Rational Numbers

As in the previous example, we use the common denominator 12 for $\frac{1}{4}$ and $\frac{1}{3}$.

To subtract the fractions one from the other, simply **subtract the numerators** and then place the difference over the **common denominator**.



$\frac{1}{3} = \frac{4}{12}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{12}$

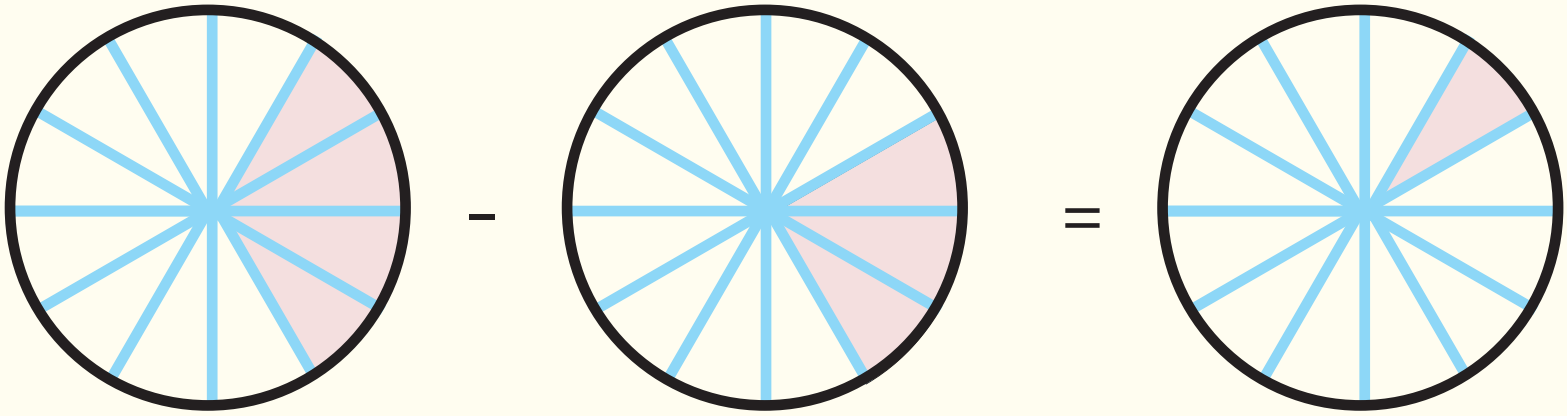
$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{4-3}{12} = \frac{1}{12}$$

THE RATIONAL NUMBERS
and their operations

Subracting Rational Numbers

If you noticed, in this example, we subtracted the smaller fraction $\frac{1}{4}$ from the larger one $\frac{1}{3}$ and obtained a **positive** answer of $\frac{1}{12}$.

$$\frac{1}{3} - \frac{1}{4} = \frac{4}{12} - \frac{3}{12} = \frac{4-3}{12} = \frac{1}{12}$$



$\frac{1}{3} = \frac{4}{12}$ $\frac{1}{4} = \frac{3}{12}$ $\frac{1}{12}$

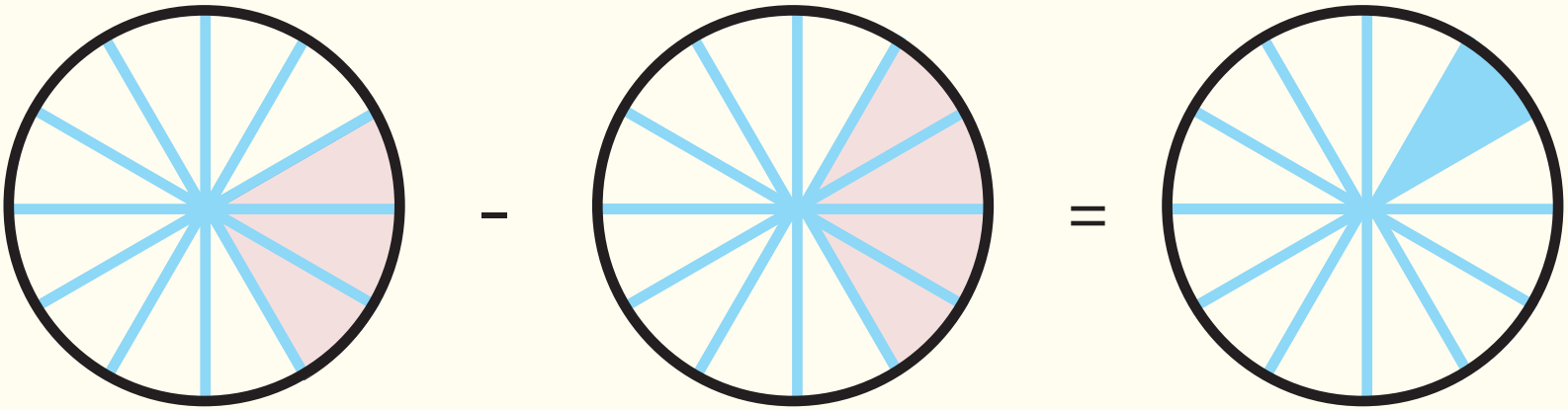
THE RATIONAL NUMBERS
and their operations

Subracting Rational Numbers

What would happen if we subtracted

the larger fraction $\frac{1}{3}$ from the smaller one $\frac{1}{4}$?

$$\frac{1}{4} - \frac{1}{3} = \frac{3}{12} - \frac{4}{12} = \frac{3-4}{12} = \frac{-1}{12}$$



$\frac{1}{4} = \frac{3}{12}$ $\frac{1}{3} = \frac{4}{12}$ $\frac{-1}{12}$

From three pieces remove four pieces, leaving one negative piece.

where the pink unit piece indicates a positive value

where the blue unit piece indicates a negative value

THE RATIONAL NUMBERS
and their operations

Subtracting Rational Numbers What to do

$$\left(\frac{p}{q}\right) - \left(\frac{m}{n}\right) := \frac{p \cdot n - m \cdot q}{q \cdot n}$$

formally

where p, m, q, n are integers.

$q, n \neq 0$

Step 1 is to find a common denominator for both fractons.
The smallest one, called the lowest common denominator (LCD), is the best choice - but not the only choice.

$q \cdot n$ is one possible choice for the above two fractions because both denominators q and n are present.

Recall that, to be able to subtract $\frac{1}{3} - \frac{1}{4}$, we needed a $4 \cdot 3 = 12$ piece pie.

Step 2 is to find equivalent fractions to the original ones, BOTH having the chosen common denominator.

$$\left(\frac{p}{q}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) \qquad \left(\frac{m}{n}\right) = \left(\frac{m \cdot q}{n \cdot q}\right)$$

NOTE $q \cdot n = n \cdot q$

THE RATIONAL NUMBERS
and their operations

Subtracting Rational Numbers What to do

$$\left(\frac{p}{q}\right) - \left(\frac{m}{n}\right) := \frac{p \cdot n - m \cdot q}{q \cdot n}$$

formally

p, m, q, n are integers.

$q \; n \neq 0$

Step 3 is to SUBTRACT the equivalent fractions together, by SUBTRACTING THE NUMERATORS and placing the difference over the chosen common denominator.

The equivalent fractions:

$$\left(\frac{p}{q}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) \qquad \left(\frac{m}{n}\right) = \left(\frac{m \cdot q}{n \cdot q}\right)$$

NOTE $q \cdot n = n \cdot q$

Performing the subtraction:

$$\left(\frac{p}{q}\right) - \left(\frac{m}{n}\right) = \left(\frac{p \cdot n}{q \cdot n}\right) - \left(\frac{m \cdot q}{n \cdot q}\right) = \frac{p \cdot n - m \cdot q}{q \cdot n}$$

NOTE $q \cdot n = n \cdot q$

THE RATIONAL NUMBERS
and their operations

Subtracting Rational Numbers

$$\left(\frac{p}{q}\right) - \left(\frac{m}{n}\right) := \frac{p \cdot n - m \cdot q}{q \cdot n}$$

formally

p, m, q, n are integers.

$q \; n \neq 0$

EXAMPLE $\left(\frac{3}{10}\right) - \left(\frac{-4}{7}\right)$ What to do

Step 1 is to find a common denominator. The smallest one, called the lowest common denominator (LCD), is the best choice - but not the only choice.

$10 \cdot 7$ is one possible choice for the above two fractions because both denominators 10 and 7 are present.

Is this the lowest common denominator?

EXAMPLE continued

What is the smallest number into which both 10 and 7 fit?

Finding LCD's

To find the lowest common denominator (LCD) from 10 and 7 we proceed as follows:

1. Factor both denominators into primes (i.e. until they can't be factored further). $10 = 2 \cdot 5$ 7 is a prime number.

2. Collect all of the ***distinct*** factors in both denominators and take the ***maximum number of each*** appearing in either one.

distinct factors in both 2 , 5 , 7

maximum number in each: **one factor of** 2 ; **one factor of** 5 ;
one factor of 7

3. Multiply together the ***maximum number of each*** **distinct factor** appearing in either denominator. This is the **LCD**.

LCD for $\frac{3}{10}$ and $\frac{-4}{7}$ is: $2 \cdot 5 \cdot 7 = 70$

EXAMPLE continued

LCD for $\frac{3}{10}$ and $\frac{-4}{7}$ is: $2 \cdot 5 \cdot 7 = 70$

Step 2 is to find equivalent fractions to the original ones, BOTH having the chosen common denominator.

$$\left(\frac{3}{10}\right) = \left(\frac{3 \cdot 7}{10 \cdot 7}\right) \qquad \left(\frac{-4}{7}\right) = \left(\frac{(-4) \cdot 10}{7 \cdot 10}\right)$$

Step 3 is to ADD the equivalent fractions together, by ADDING THE NUMERATORS and placing the sum over the chosen common denominator.

$$\left(\frac{3}{10}\right) - \left(\frac{-4}{7}\right) = \left(\frac{3 \cdot 7}{10 \cdot 7}\right) - \left(\frac{(-4) \cdot 10}{7 \cdot 10}\right)$$

$$= \frac{21 - (-40)}{70} = \frac{61}{70}$$

THE RATIONAL NUMBERS
and their operations

Subtracting Rational Numbers

EXAMPLE $\left(\frac{3}{2}\right) - \left(\frac{12}{13}\right)$ What *not* to do

$$\left(\frac{3}{2}\right) - \left(\frac{12}{13}\right) \neq \left(\frac{3}{\cancel{2}_{1}}\right) - \left(\frac{\cancel{12}^{6}}{13}\right)$$

DO NOT cancel common factors across a $-$ sign.

$$\neq \frac{3 - 6}{1 - 13} = \frac{-3}{-12}$$

DO NOT subtract numerators without a common denominator.

THE RATIONAL NUMBERS
and their operations

Addendum: Mixed Numbers

Mixed numbers are combinations (actually additions) of whole numbers and fractions.

EXAMPLES

$1\frac{1}{2}$ is read as: one and a half

is interpreted as: $1 + \frac{1}{2}$

is equal to: $1 + \frac{1}{2} = \frac{1}{1} + \frac{1}{2} = \frac{1 \cdot 2}{1 \cdot 2} + \frac{1}{2} = \frac{3}{2}$

$-3\frac{5}{8}$ is read as: minus (or negative) three and five eighths

is interpreted as: $-\left(3 + \frac{5}{8}\right)$

is equal to: $-\left(3 + \frac{5}{8}\right) = -\left(\frac{3}{1} + \frac{5}{8}\right) = -\left(\frac{3 \cdot 8}{1 \cdot 8} + \frac{5}{8}\right) = -\frac{29}{8}$