

USING LOGARITHMS to SOLVE FINANCIAL PROBLEMS

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000 and have \$15000 after 3 years. Suppose also that your banker offers you semi-annual compounding on your investment.

What annual interest rate would be necessary in order for your banker to fulfill your request?

FORMULA FOR COMPOUND INTEREST

$$S = P \cdot (1+i)^n$$

where:

- P = original investment
 - i = interest rate per compounding period
 - n = number of compounding periods
 - S = value at maturity
-
-

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000 and have \$15000 after 3 years. Suppose also that your banker offers you semi-annual compounding on your investment.

What annual interest rate would be necessary in order for your banker to fulfill your request?

Solution:

$$S = P \cdot (1+i)^n$$

where:

$P = \$10,000$ = original investment

i = interest rate semi-annually

$n = 2 \cdot 3 = 6$ = number of compounding periods

$S = \$15,000$ = value at maturity

We must solve the following equation for the unknown i .

$$15000 = 10000 (1+i)^6$$

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000 and have \$15000 after 3 years. Suppose also that your banker offers you semi-annual compounding on your investment.

What annual interest rate would be necessary in order for your banker to fulfill your request?

Solution (continued):

$$15000 = 10000 (1+i)^6$$

Divide both sides by 10000.

$$1.5 = (1+i)^6$$

Take the logarithm (to the base 10) of both sides of the equation.

$$\log 1.5 = \log (1+i)^6$$

Apply the log of a power property.

$$\log 1.5 = 6 \log (1+i)$$

Evaluate the log on the left and divide both sides by 6.

$$\frac{0.1760912591}{6} = 0.029348543 = \log_{10} (1+i)$$

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000 and have \$15000 after 3 years. Suppose also that your banker offers you semi-annual compounding on your investment.

What annual interest rate would be necessary in order for your banker to fulfill your request?

Solution (continued):

$$0.029348543 = \log_{10}(1+i) \quad \text{Apply the definition of log.}$$

$$10^{0.029348543} = (1+i) \quad \text{Solve for } i.$$

$$1.069913194 = (1+i)$$

$$1.069913194 - 1 = i \quad \text{OR} \quad 0.069913194 = i$$

In order to get \$15,000 after 3 years, the semi-annual interest rate would have to be: $i = 7\%$

Answer: The annual interest rate would have to be 14% .

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000.

If your banker offers you a special GIC interest rate of 5.0% with only annual compounding on your investment,

how long would it take before you doubled your initial investment?

FORMULA FOR COMPOUND INTEREST

$$S = P \cdot (1+i)^n$$

where:

- P = original investment
 - i = interest rate per compounding period
 - n = number of compounding periods
 - S = value at maturity
-
-

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000.

If your banker offers you a special GIC interest rate of 5.0% with only annual compounding on your investment,

how long would it take before you doubled your initial investment?

Solution:

$$S = P \cdot (1+i)^n$$

where:

$P = \$10,000$ = original investment
 $i = .05$ = interest rate annually
 n = number of years
 $S = \$20,000$ = value at maturity

We must solve the following equation for the unknown n .

$$20000 = 10000 (1 + .05)^n$$

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000.

If your banker offers you a special GIC interest rate of 5.0% with only annual compounding on your investment,

how long would it take before you doubled your initial investment?

Solution (continued):

$$20000 = 10000 (1 + .05)^n$$

Divide both sides by 10000.

$$2 = (1.05)^n$$

Take the logarithms to the base 10 of both sides of the equation.

$$\log 2 = \log (1.05)^n$$

Apply the log of a power property.

$$\log 2 = n \log (1.05)$$

Evaluate the logs on the left and right.

$$0.30102999566 = n \cdot 0.02118929907$$

SOLVING FINANCIAL PROBLEMS USING LOGARITHMS

Suppose you want to invest \$10000.

If your banker offers you a special GIC interest rate of 5.0% with only annual compounding on your investment,

how long would it take before you doubled your initial investment?

Solution (continued):

$$0.30102999566 = n \cdot 0.02118929907 \quad \text{Solve for } n.$$

$$\frac{0.301029996}{0.0211892991} = n$$

$$14.2067 = n$$

Answer: It would take 14 years and 2 ½ months.

