

The History of Numbers

from Babylon

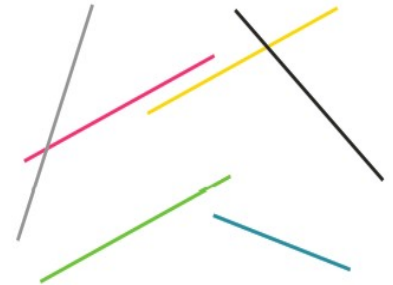
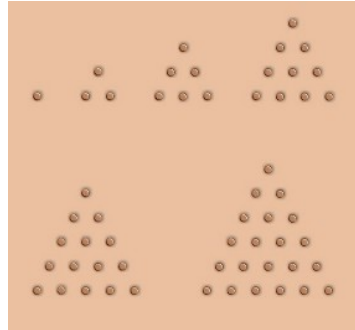
to

Pythagoras

to

Euclid

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Since the time of the ancient Egyptians, Chinese, Sumerians, Assyrians, Babylonians, Phoenicians, and the peoples of the Indian subcontinent, the notion of a number and its individual symbolic textual notations had been used as a practical measure for calculating magnitudes.

To the Greek philosopher, Pythagoras, who lived in the 6th century B.C., magnitudes were better represented by pebbles (or *calculi*) in various geometric forms. This led to a more abstract notion of number in the Ancient Greek mathematical community. Numbers were more than unique textual symbols which the Babylonians and other ancient civilizations had used for trading and record keeping. To them, numbers were mathematical objects abstracted into piles of amorphous pebbles and were distinguished one from another by quantity and form, not by symbol.

By the time of Euclid in the early 4th century B.C., there had been a significant change even from this philosophical abstraction. Magnitudes were no longer associated with numbers or pebbles (as 0-dimensional points in space), but with line segments drawn as 1-dimensional geometric objects whose quantity was measured by length. In Euclid's treatise, *The Elements*, even the integers themselves were represented by *line segments with finite length*, still retaining the property of discreteness, but without direction or orientation as fully extended lines.

In this interim period, two discoveries in the 5th century B.C. had perturbed the mathematical community of the Ancient Greeks. The first was the discovery, by Hippasus of Metapontum, of numbers which were incommensurable, meaning not able to be calculated. The second was the conundrum of Zeno's Paradox that a runner who is able to cover half the remaining distance to the target in discrete time segments can mathematically never reach his goal. This led to the realization that the discrete infinitesimal subdivision of space and time led to the absurdity that continuous motion is impossible.

Nonetheless, throughout both Pythagorean and Hellenic mathematics, there was a parallel concept of "continuous magnitudes" (quite apart from the notion of the discrete number) which had to be treated using the geometric method. It was this perspective that ultimately led to the geometry of the "real line" or the *Continuum* (mathematically proved only millenia later in the latter half of the nineteenth century). It was, however, the notion of the *continuous magnitude* which, almost 2000 years after Euclid, was used by Descartes to conceive of, what is now called, the *Cartesian Plane* with its grid of two continuous real line axes and its points represented as numbered pairs.